

# Effect of Suspended Solid Particles on the Properties in Cascade Flow

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An approximate method for calculating the flow properties of gas-particle mixture flowing over blades in a cascade is developed. The momentum and heat transfer between the gas and particles as well as compressibility effects are taken into consideration. The effect of particle concentration, mean diameter, particle material density, particle inlet velocity and temperature on the gas particle flow properties are investigated.

## Nomenclature

$A$  = cross-sectional area of stream tube ( $\text{ft}^2$ )  
 $A'$  = nondimensional cross-sectional area of stream tube  
 $\alpha$  = the ratio of the mass flow rate of particles to the total mass flow rate of the gas and particle mixture  
 $A_p$  = mean particle projected area ( $\text{ft}^2$ )  
 $A_s$  = mean particle surface area ( $\text{ft}^2$ )  
 $C_d$  = particles drag coefficient  
 $C_{pp}$  = specific heat of solid particles material ( $\text{Btu/lb}^\circ\text{R}$ )  
 $C_{pg}$  = gas specific heat at constant pressure ( $\text{Btu/lb}^\circ\text{R}$ )  
 $\delta s$  = distance along airfoil contour between two successive points (ft)  
 $dt$  = time in which the particle travels a distance  $\delta$  (sec)  
 $d_p$  = particle mean diameter (ft)  
 $F$  = coefficient  
 $f$  = correction factor to Stokes drag formula  
 $g$  = gravity constant  
 $h$  = coefficient of heat transfer between the particles and the gas ( $\text{Btu/hr ft}^2^\circ\text{R}$ )  
 $J$  = mechanical equivalent of heat  
 $k_g$  = coefficient of conductivity for the gas ( $\text{Btu/hr ft}^\circ\text{R}$ )  
 $m$  = mass of one particle (lb)  
 $Nu$  = Nusselt number  
 $P$  = gas particle suspension pressure ( $\text{lb/ft}^2$ )  
 $P'$  = nondimensional gas particle suspension pressure  
 $p$  = pressure of gas-only flow ( $\text{lb/ft}^2$ )  
 $p'$  = nondimensional gas-only flow pressure  
 $Pr$  = Prandtl number  
 $R_g$  = gas constant  
 $Re$  = Reynolds number  
 $\rho$  = the gas-only flow density ( $\text{lb/ft}^3$ )  
 $\rho'$  = the gas-only flow nondimensional density  
 $\rho_g$  = the gas density in gas particle flow ( $\text{lb/ft}^3$ )  
 $\rho_p$  = the particles density ( $\text{lb/ft}^3$ )  
 $\rho_p'$  = the nondimensional particles density  
 $\rho_p$  = solid particle material density ( $\text{lb/ft}^3$ )  
 $s$  = distance along airfoil contour (ft)  
 $T_0$  = total temperature of flow ( $^\circ\text{R}$ )  
 $T$  = the gas-only flow temperature ( $^\circ\text{R}$ )  
 $T'$  = the nondimensional temperature of gas-only flow  
 $T_g$  = the gas temperature in gas particle flow ( $^\circ\text{R}$ )  
 $T_g'$  = the nondimensional gas temperature in gas particle flow  
 $T_p$  = particle temperature ( $^\circ\text{R}$ )

$T_p'$  = nondimensional particle temperature  
 $t$  = time (sec)  
 $u$  = gas-only flow velocity, fps  
 $u'$  = gas-only flow nondimensional velocity  
 $u_g'$  = nondimensional gas velocity in gas particle flow  
 $u_g$  = gas velocity in gas particle flow, fps  
 $u_p$  = particle velocity, fps  
 $u_p'$  = nondimensional particle velocity  
 $W$  = mass flow rate of gas particle mixture or mass flow rate of gas-only flow ( $\text{lb/sec}$ )  
 $x, y$  = Cartesian coordinates

## Subscripts

$g$  = gas  
 $p$  = particle  
 $1$  = condition at starting point of the stream tubes

## Introduction

THE study of gas particle flow over compressor or turbine blades is an area of recent interest. This is, in part, due to the development of high energy propellants for combustion system in which the product of combustion may partially consist of finely divided particles. For example, turbines which are invariably present in nuclear and liquid propellant rockets, are subjected to gas particle flow. Air-breathing engines operating in desert areas, where the inlet airflow may contain sand particles, provide another example.

The flow of gas particle-suspension<sup>1-4</sup> involves gas particle interaction through viscous drag and heat transfer. This interaction causes changes in the properties of gas and particles. These particles are swept through the cascade nozzle by gas flow, lagging behind the gas in velocity and temperature. Both effects lead to the deterioration in the performance of compressor or turbine. Furthermore, the particles may cause blade erosion and consequently failure. Even when the erosion is not severe enough to cause rupture, changes of blade geometry affect cascade performance.

Blade erosion depends upon several factors, some of which are: the properties of the blade material; the total mass of impinging particles; the particle speed and the angle of attack; and the temperature of both the gas and the particles at the blade surface. Before approaching these problems it is necessary to determine the way in which the gas particle properties vary with the distance along the surface of the blade.

## Method of Solution

A numerical solution to the problem of gas particle flow over blades in cascade is obtained, utilizing the known solution for isentropic flow without particles over the same blade.

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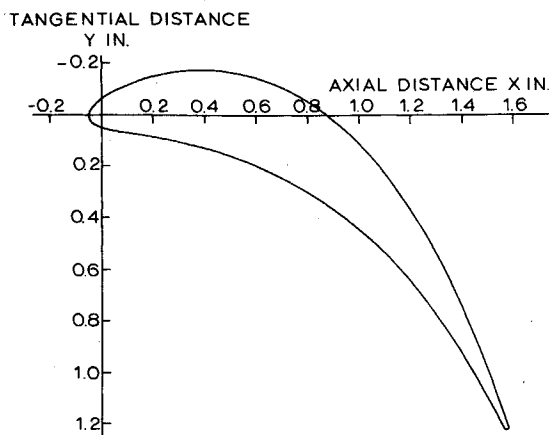


Fig. 1 Airfoil shape.

If the flow of a gas past a cascade is considered, then the velocity or the pressure distribution over any blade section can be determined theoretically or experimentally. Using a given velocity distribution, together with the inlet conditions of the gas, (and assuming isentropic expansion through the cascade nozzle), the temperature, Mach number, pressure and density distributions over the airfoil are determined.

It is assumed that two stream tubes exist in the flow: one at the suction side, the other at the pressure side of the airfoil. The gas mass flow rate through each tube is taken as  $W$ . The calculated gas condition at the airfoil surface can then be used to determine the nondimensional area of the stream tube. The stream tubes of known nondimensional cross-sectional area distribution are considered to be one-dimensional nozzles. It is further assumed that a mixture of air and particles having a total rate of mass flow  $W$  will pass through each stream tube. Governing equations for the gas-particle flow can be formulated, given these conditions. The equations of continuity, momentum, energy, state, convective heat transfer between particles and gas, and the equation for drag on the particle due to its relative motion with respect to the gas, are solved numerically on a computer. The properties of the gas particle flow at each point of the airfoil are then determined.

The step by step numerical solution of the governing equations is obtained by dividing the distance along the airfoil contour into a number of small increments. As a consequence of the gas velocity being zero at the blade stagnation point, the nondimensional stream tube cross-sectional area would be infinite at the leading edge. Because of this the construction of the stream tubes is started at a small distance from the leading edge.

In deriving all the governing equations, reference is made to one of the stream tubes. However the same arguments are equally valid for the stream tube on the other side of the airfoil.

To derive the governing equations for gas particle flow, the following assumptions are made:

- 1) The flow in the stream tube for sufficiently small steps, and relatively small blade camber and stagger angle, is assumed to be steady, and can be considered one dimensional.
- 2) The solid particles are uniformly distributed over each cross section. The gas properties are uniform and the particle conditions, though different from the gas conditions, are also uniform at each cross section.
- 3) The solid particles are spheres of uniform diameter and physical properties.
- 4) The solid particles, owing to their small size and high thermal conductivity (relative to that of the gas), are assumed to be at a uniform temperature.
- 5) The volume occupied by the particles is neglected.
- 6) The particles

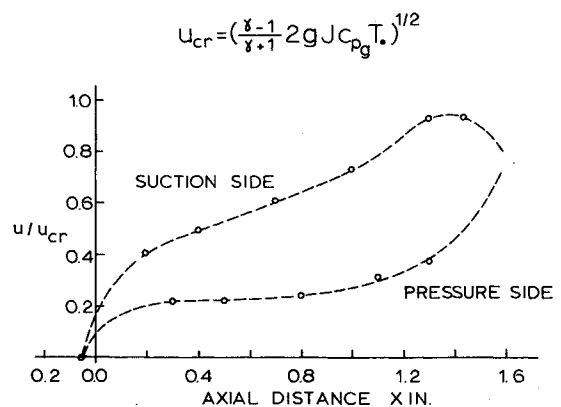


Fig. 2 Experimental velocity distribution.

are far enough apart so that the motion of each one of them is not affected by the motion of its neighbors. 7) The drag on the particles is mainly due to the differences between the mean velocities of particles and the gas stream. It is expected that the smallest of the solid particles consists of millions of molecules each; hence the velocity of each solid particle due to its thermal state is extremely low, and the particle Brownian motion does not contribute to the pressure of the system. 8) The heat transfer between the gas and the solid particles is by convection only, and is basically due to their mean temperature difference. 9) The particles do not interact either with one another or with the boundaries. 10) At the solid boundaries (blade surface) friction forces are zero, and there is no heat transfer. 11) The gas is assumed perfect, and its specific heat as well as that of the solid particle material are assumed constant. 12) The gas is inviscid except for the drag it exerts on the solid particles.

### Construction of Suction and Pressure Sides Stream Tubes

#### 1. Distance along airfoil contour between successive points

The data and dimensions of the airfoil and cascade used in the theoretical discussion is given in Ref. 5. The airfoil contour is defined by means of coordinate values  $x$  and  $y$  (Fig. 1). A quadratic curve is fitted through each three successive points on the contour, the resulting equation is used to calculate the length  $\delta_s$  along the blade contour between each two points.

#### 2. Gas flow properties on airfoil surface

The velocity ratio distribution ( $u/u_{cr}$ ) on the airfoil surface (Fig. 2) is taken from test results of Ref. 5. Assuming the inlet conditions to the stream tube to be  $T_0$  and  $p_1$ , the velocity distribution can be calculated from Fig. 2. Values of the nondimensional temperature, pressure, velocity and

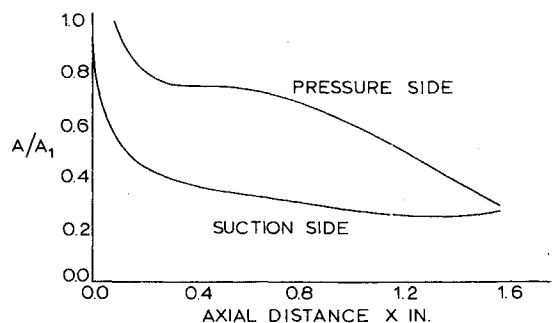


Fig. 3 Nondimensional area distribution of the stream tubes.

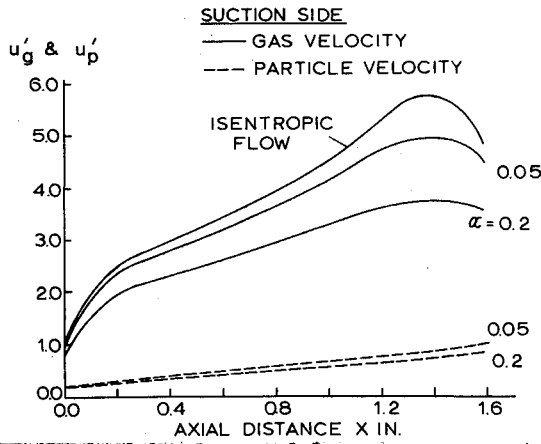


Fig. 4 Effect of the change of the particle concentration on  $u'_g$  and  $u'_p$ .

density are defined as follows:

$$T' = T/T_1; \quad p' = p/p_1; \quad u' = u/u_1; \quad \rho' = \rho/\rho_1 \quad (1)$$

These nondimensional parameters can be calculated at each point by applying the energy equation and the perfect gas relations. The nondimensional area distribution of the stream tube of the gas flow on the airfoil surface (Fig. 3) can be calculated from the continuity equation as

$$A' = A/A_1 = 1/\rho'u' \quad (2)$$

### 3. Rate of change of stream tube nondimensional cross-sectional area

For a gas flow passing through a stream tube it is possible to list the momentum, energy, continuity equations and the equation of state in the differential form as

$$\rho u du/ds = -(dp/ds)g \quad (3)$$

$$gJC_{p0}(dT/ds) + u du/ds = 0 \quad (4)$$

$$(1/\rho)dp/ds + (1/u)du/ds + (1/A)dA/ds = 0 \quad (5)$$

$$(1/p)dp/ds = (1/\rho)d\rho/ds + (1/T)dT/ds \quad (6)$$

Substitution of the nondimensional parameters given by Eqs. (1) and (2) into Eqs. (3–6) and solving the resulting equations simultaneously for  $dA'/ds$  as function of  $du'/ds$  results in the equation

$$dA'/ds = [(\rho_1 u_1^2 / g p_1) u' A' / T' - u_1^2 / g J C_{p0} T_1 \cdot u' A' / T' - A' / u'] du' / ds \quad (7)$$

This is the expression for the rate of change of the stream tube nondimensional cross-sectional area as a function of the

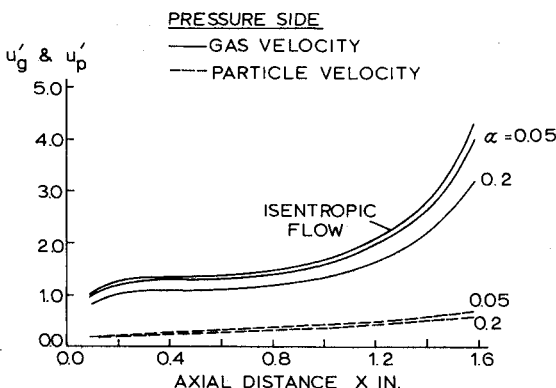


Fig. 5 Effect of the change of the particle concentration on  $u'_g$  and  $u'_p$ .

rate of change of gas nondimensional velocity, gas properties and conditions at the stream tube starting point.

### Gas Particle Flow Governing Equations

It is more convenient to use the gas particle flow governing equations in nondimensional form. To accomplish this, the following parameters are defined: nondimensional pressure of gas particle flow ( $P'$ ); nondimensional gas velocity, temperature and density ( $u'_g, T'_g$  and  $\rho'_g$ ); and nondimensional particle velocity, temperature and density ( $u'_p, T'_p$  and  $\rho'_p$ ). These are expressed as

$$\begin{aligned} P' &= P/P_1, \quad u'_g = u_g/u_{g1}, \quad T'_g = T_g/T_{g1} \\ \rho'_g &= \rho_g/\rho_{g1}, \quad u'_p = u_p/u_{g1} \\ \rho'_p &= \rho_p/\rho_{g1}, \quad T'_p = T_p/T_{g1} \end{aligned} \quad (8)$$

### Derivation of an Expression for the Drag over the Particles

When a particle is introduced into a stream tube of flowing gas, the interaction between the gas and the particle creates a drag force on the particle. The force is only present when there is relative motion between the gas and the particle. The drag force is given by

$$\frac{md^2s}{dt^2} = C_d [\frac{1}{2} \rho_g (u_g - u_p)^2 A_p] \quad (9)$$

where  $A_p = \pi/4 d_p^2$  and  $C_d$  is the corrected drag coefficient.

Gilbert<sup>6</sup> introduces a correction factor  $f$  to the drag formula given by Stokes in the case of spherical particle moving in gas as

$$f = (28Re^{-0.85} + 0.48)/24Re^{-1.0} \quad (10)$$

where the Reynolds number  $Re$  is defined as

$$Re = \frac{\rho_g d_p (u_g - u_p)}{\mu_g} \quad (11)$$

Utilizing Eqs. (10) and (11), the corrected drag coefficient is

$$C_d = \frac{24f}{Re} = 24\mu_g / [\rho_g d_p (u_g - u_p)] f \quad (12)$$

Substitution for  $A_p$  and  $C_d$  into Eq. (9) and the use of the nondimensional forms from Eq. (8) admits the nondimensional drag equation

$$u'_p du'_p / ds = F / u_{g1} (u'_g - u'_p) \quad (13)$$

where,

$$F = (18\mu_g / \bar{\rho}_p d_p^2) f \quad (14)$$

### Equation of Heat Transfer for Flow Over Spherical Particle

An empirical equation for the Nusselt number that correlates data for Reynolds numbers ranging up to 70,000 and Prandtl numbers from 2.6–400 is given in Ref. 6 for the case of heat convection from the surface of a spherical particle immersed in a fluid stream. This equation is

$$Nu = 2 + 0.6Re^{0.5} Pr^{1/3} \quad (15)$$

From the definition of the Nusselt number Eq. (15) can be rearranged as

$$h = k_g / d_p (2 + 0.6Re^{0.5} Pr^{1/3}) \quad (16)$$

The heat exchange between the particle and the gas for a time  $dt$  is given by

$$hA_s(T_p - T_g)dt = mC_{pp}(-dT_p) \quad (17)$$

where  $dt$  is the time in which the particle travels a distance  $ds$  and

$$A_s = \pi d_p^2; \quad m = \pi/6 d_p^3 \bar{\rho}_p \quad (18)$$

Since  $u_p = ds/dt$  a substitution of  $u_p$  into Eq. (17) gives

$$u_p dT_p/ds = -[hA_s(T_p - T_g)/mC_{pp}] \quad (19)$$

Substitution of the expression for  $hA_s$  and  $m$  from Eqs. (16) and (18) into Eq. (19), making use of definitions in Eq. (8), the following nondimensional heat-transfer equation is

$$u_p' dT_p'/ds = -6k_g/\bar{\rho}_p C_{pp} d_p^2 u_{g1} (T_p' - T_g') \times (2 + 0.6Re^{0.5}Pr^{1/3}) \quad (20)$$

### Momentum Equation

The definition of the particle concentration  $\alpha$  can be expressed as

$$\alpha = W_p/(W_p + W_g) = W_p/W \quad (21)$$

since

$$W_p = \alpha W \quad \text{and} \quad W_g = (1 - \alpha)W \quad (22)$$

The momentum equation for gas particle mixture is then

$$dP/ds = -(W/gA)[(1 - \alpha)du_g/ds + \alpha du_p/ds] \quad (23)$$

The nondimensional form of Eq. (23) can be obtained by substituting Eq. (8) in the above expression to get,

$$dP'/ds = -(\rho_{g1}u_{g1}^2/gP_1A')\{du_g'/ds + [\alpha/(1 - \alpha)] \times du_p'/ds\} \quad (24)$$

### The Energy Equation

The conservation of energy is expressed as

$$[JC_{pg}dT_g/ds + (u_g/g)du_g/ds] \cdot (1 - \alpha)W + [JC_{pp}dT_p/ds + (u_p/g)du_p/ds] \cdot \alpha W = 0 \quad (25)$$

Substitution of Eq. (8) into Eq. (25) in nondimensional form allows  $dT_g'/ds$  to be expressed as

$$dT_g'/ds = (T_g'/A')dA'/ds - [\alpha/(1 - \alpha)] \cdot \rho_{g1}u_{g1}^2/gP_1 \cdot u_g'du_p'/ds + u_g'du_g'/ds \cdot (T_g'/u_g'^2 - \rho_{g1}u_{g1}^2/gP_1) \quad (26)$$

### Equation of State for the Gas

The equation of state for the gas is

$$P = \rho_g R_g T_g \quad (27)$$

Substitution for  $\rho_g$  from the continuity equation for gas results in

$$P = (1 - \alpha)W/u_g A \cdot R_g T_g \quad (28)$$

Differentiation of Eq. (28) with respect to  $s$  gives,

$$dP/ds = (1 - \alpha)R_g W[(1/Au_g)dT_g/ds - (T_g/A^2u_g)dA/ds - (T_g/u_g^2A)du_g/ds] \quad (29)$$

Equations (23) and (29) can be solved for  $dT_g/ds$  and put into nondimensional form as

$$dT_g'/ds = (T_g'/A')dA'/ds - [\alpha/(1 - \alpha)]u_{g1}^2/gR_gT_{g1} \times u_g'du_p'/ds + [T_g'/u_g' - (u_{g1}^2/gR_gT_{g1})u_g']du_g'/ds \quad (30)$$

Elimination of  $dT_g'/ds$  between Eqs. (26) and (30), admits  $du_g'/ds$  as function of  $\rho du_p'/ds$ ,  $dT_p'/ds$  and  $dA'/ds$ ; that is,

$$du_g'/ds = \{-(T_g'/A')dA'/ds - \alpha/(1 - \alpha) \cdot \{(C_{pp}/C_{pg})dT_p'/ds + [(u_{g1}/gJC_{pg}T_{g1})u_p' - (\rho_{g1}u_{g1}^2/gP_1)u_g']du_p'/ds\}/[u_g'(u_{g1}^2/gJC_{pg}T_{g1}) + T_g'/u_g' - \rho_{g1}u_{g1}^2/gP_1]\} \quad (31)$$

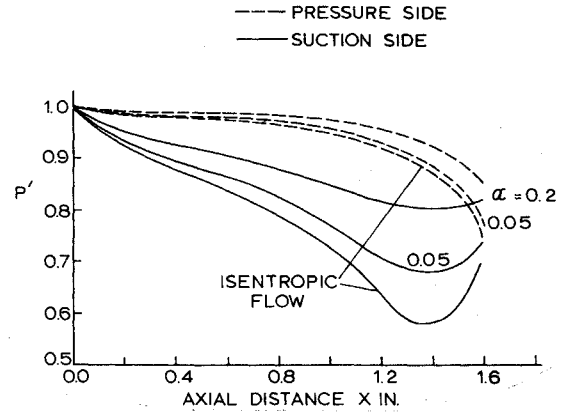


Fig. 6 Effect of the change of the particle concentration on  $P'$ .

### Continuity Equations for Gas and Particles

Using the expression for  $W_p$  and  $W_g$  from Eq. (22) together with Eq. (8), the nondimensional continuity equations for gas and particles can be written respectively as

$$\rho_g' u_g' A' = 1 \quad (32)$$

$$\rho_p' u_p' A' = \alpha/(1 - \alpha) \quad (33)$$

### Procedure for Numerical Solution of Nondimensionalized Governing Equations for the Gas Particle Flow

Equations (13, 20, 24, 26, and 31-33) are the nondimensionalized governing equations which describe the gas particle flow over compressor or turbine blades. This system of nonlinear differential equations and algebraic equations were solved numerically. The increments in various parameters in a short length  $\delta s$  of the stream tube are considered for a cross section area in the stream tube at which the gas and particle conditions are known. The derivatives  $du_p'/ds$  and  $dT_p'/ds$  may be obtained from Eqs. (13) and (20) while  $dA'/ds$  is known from the stream tube geometry. Substitution of these values into Eq. (31) gives  $du_g'/ds$ , and using Eqs. (24) and (26),  $dT_g'/ds$  and  $dP'/ds$  may be obtained. These slopes were assumed to apply over a small length  $\delta s$ , enabling estimates of gas and particle conditions at  $(s + \delta s)$  to be obtained. Equations (32) and (33) then can be used to determine the value of  $\rho_g'$  and  $\rho_p'$  at  $(s + \delta s)$ . The transport properties of the gas may be allowed to vary by expressing them as a function of gas temperature. An iterative procedure gives average values of  $F, k_g, Pr, \mu_g$  for the interval, and the process may be continued until successive sets of conditions agree to the required accuracy. The pro-

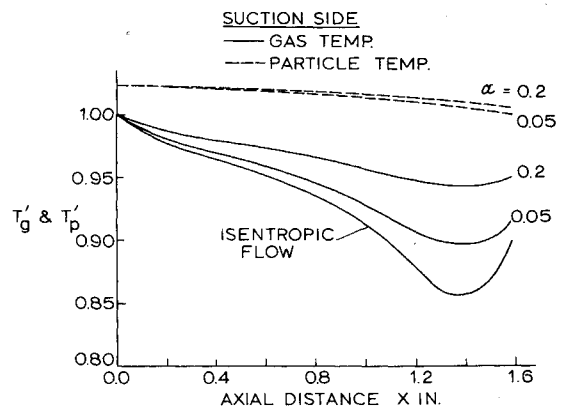


Fig. 7 Effect of the change of the particle concentration on  $T_g'$  and  $T_p'$ .

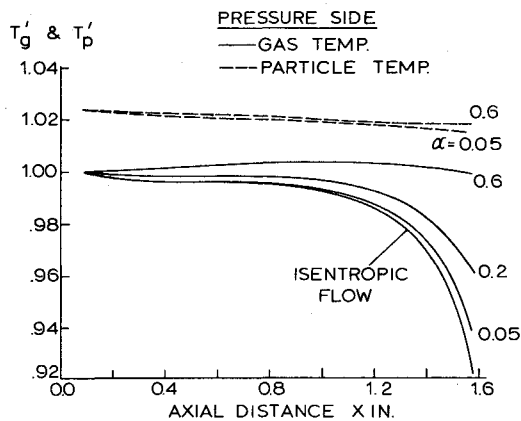


Fig. 8 Effect of the change of the particle concentration on  $T_g'$  and  $T_p'$ .

cedure is then repeated for the next interval and so on. This was the general method adopted to solve specific cases, and the calculations were performed on a digital computer.

### Numerical Example

Different numerical examples are performed to show the effects of a change in particle concentration  $\alpha$ , particle mean diameter  $d_p$ , solid particle material density  $\bar{\rho}_p$ , particle inlet velocity and temperature on the gas particle flow properties.<sup>7</sup> The gas inlet condition was taken to be

$$T_0 = 65^\circ\text{F}, P_1 = 29.4 \text{ lb/in.}^2$$

For the purposes of this example the suspension was assumed to consist of aluminum particles of spherical shape, i.e.,  $C_{pp} = 0.215 \text{ Btu/lb}^\circ\text{R}$  and density  $\bar{\rho}_p = 168.5 \text{ lb/ft}^3$ .

In the examples treated, the starting point was taken as the point where  $u/u_{cr}$  is equal to 0.15 both on the pressure and suction sides of the blade. This corresponds to the point  $x = 0.085$  on the pressure side and  $x = 0.0$  on the suction side. It can be observed that the nondimensional gas velocity approaches one at the corresponding starting points on both suction and pressure sides in Figs. 4 and 5 respectively.

The resulting flow properties are shown in Figs. 4-8. Gas properties are compared with the isentropic case, to show the deviation of gas properties from isentropic conditions whenever particles are introduced in the flow. The change in flow properties due to a change of particle mean diameter is presented in Figs. 9-11. The properties of the gas particle flow are significantly affected by the change of the particle material density  $\bar{\rho}_p$ , as shown in Figs. 12 and 13.

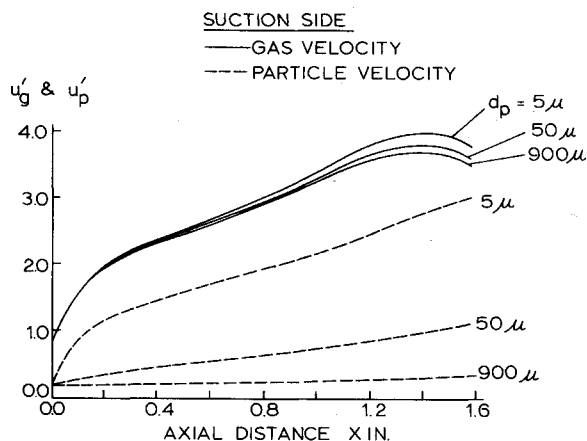


Fig. 9 Effect of the change of the particle mean diameter on  $u_g'$  and  $u_p'$ .

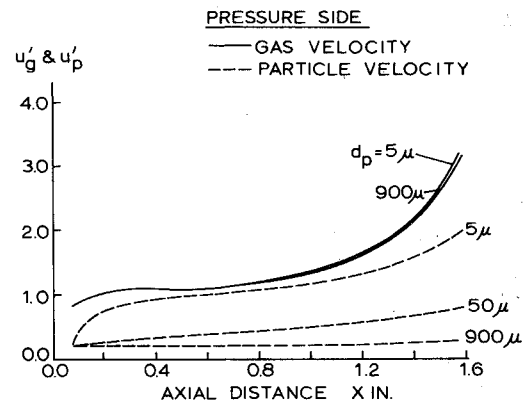


Fig. 10 Effect of the change of the particle mean diameter on  $u_g'$  and  $u_p'$ .

### Discussion

The validity of some of the assumptions made in deriving the governing equations for gas particle flow needs further discussion. The assumption that the solid particles follow the stream tube can be expected to hold only when the relative magnitude of the centrifugal force acting on the particles is small. The associated requirements for this to be true are: relatively small values of particle mean diameter, solid particle material density, particle velocity, airfoil camber and deflection angles. These flow parameters together with the particle concentration are the actual limitations on the assumption of one dimensionality of the flow. The range of validity of the analysis will decrease for relatively high values of these parameters. Also neglected are the complicated interactions which arise from the random reflections occurring near the leading edge. The magnitude of this effect is particularly difficult to assess.

The solution of the nondimensional governing equations for Reynolds numbers near zero can be determined by perturbation methods. As can be seen from the results, the effect of  $\alpha, d_p, \bar{\rho}_p, (u_{p1}/u_{g1})$  and  $(T_{p1}/T_{g1})$  on the gas particle flow properties is in general more significant on the airfoil suction side than the pressure side. This is due to the rate of change of stream tube nondimensional area being higher for the suction side.

The results show that the increase in particle concentration  $\alpha$ , causes an approximately equal decrease in both gas and particle velocities, an increase in gas-particle flow pressure, and an increase in gas and particle temperatures. However, the rise in gas temperature is much greater than that of the particles due to the heat transferred from particles to the gas. The rise in  $\alpha$  causes a slight increase in gas density and a large increase in particle density. For values of  $\alpha$  greater than 0.2 the resulting high values of particle density are not expected to be realistic. This follows from the fact that for high concentrations the particles may not follow the stream tube and hence will not reach the calculated particle density.

If the mean particle diameter is made larger, there are three main consequences. First, the particle and gas velocities become smaller, although the decrease in particle velocity is much more pronounced; secondly, the gas particle mixture pressure rises, and thirdly, for the same particle concentration  $\alpha$ , the increase in  $d_p$  implies a decrease in particle surface area and hence, a rise in particle temperature is expected owing to the reduction in the heat-transfer rate. This will produce a drop in the gas temperature, but apparently less than the corresponding increase in particle temperature. In general, changes in the flow properties arising from the increase in  $d_p$  are less than that which results from the increase of  $\alpha$ .

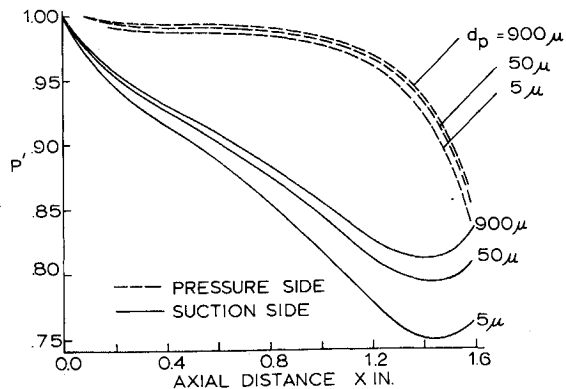


Fig. 11 Effect of the change of the particle mean diameter on  $P'$ .

The effect of increasing solid particle material density  $\bar{\rho}_p$  is to diminish slightly the gas velocity and greatly the particle velocity. This increase in  $\bar{\rho}_p$  also causes a slight increase in flow pressure. The effect on the gas temperature is slight while the particle temperature increases significantly. This may be due to the fact that for the same particle concentration with heavier particles, the number of particles per unit volume of mixture will diminish. This means that less heat will be transferred to the gas resulting in a particle temperature rise. It has been found that, in general, changes in  $\bar{\rho}_p$  are less significant in effecting flow properties than changes in  $d_p$ .

The increase in particle inlet velocity, i.e., increase in  $(u_{p1}/u_{g1})$ , has negligible effect on gas-particle flow pressure and on gas and particle temperatures. The effect on the gas velocity and density is slight; however, an appreciable increase in particle velocity and decrease in particle density results.

Increasing the inlet particle temperature, i.e., increasing  $(T_{p1}/T_{g1})$ , has negligible effect on gas and particle velocities and densities, and on flow pressure. This increase in  $(T_{p1}/T_{g1})$  causes a considerable rise in particle temperature but a slight rise in gas temperature.

In general, at the suction side of the airfoil, the gas velocity decreases near the end of the stream tube. This arises because of the increase in the stream tube nondimensional area  $A'$ . The particle velocity does not have the same trend because of its high inertia relative to the gas,

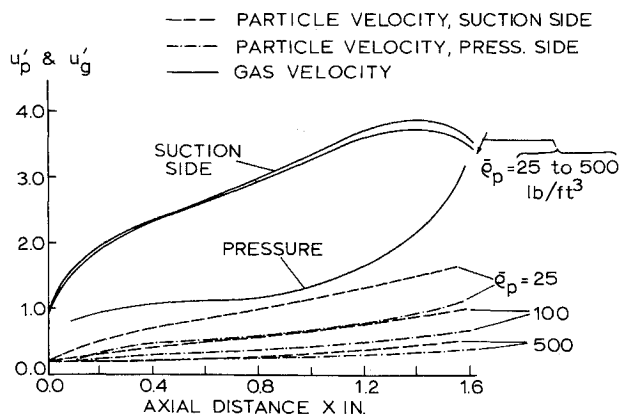


Fig. 12 Effect of the change of the particle material density on  $u_g'$  and  $u_p'$ .

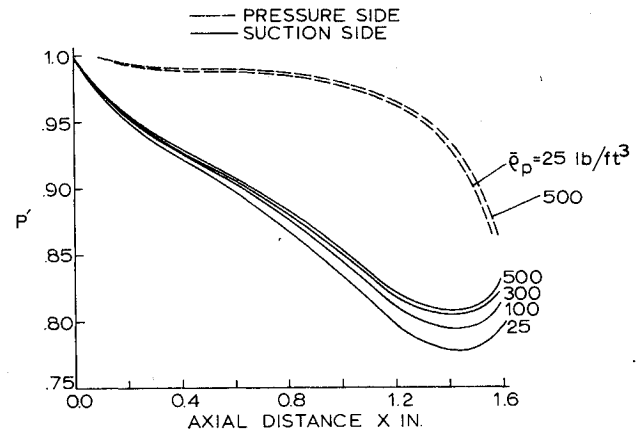


Fig. 13 Effect of the change of the particle material density on  $P'$ .

which overcomes the effect of the increase in  $A'$ . Thus particle velocity increases until it leaves the stream tube.

As shown in Fig. 2, the experimental velocity distribution on the blade surface was given from the leading edge to a point slightly upstream of the trailing edge of the blade. The pressure distribution curves for both the suction and pressure sides of the blade are shown in Figs. 6, 11 and 13. These curves show no discontinuity if they are extrapolated to the trailing edge. Similar arguments are equally valid for other flow properties such as gas temperature.

From this discussion, we can conclude that the particle concentration  $\alpha$ , is the predominant factor that substantially affects all gas particle flow properties. On the whole the changes in  $d_p$  and  $\bar{\rho}_p$  affect the particle properties more than those of the gas; and, in general, has a smaller effect than  $\alpha$  on the gas properties. The effect of  $(u_{p1}/u_{g1})$  and  $(T_{p1}/T_{g1})$  on all properties is slight, except for the velocities and temperature.

## Conclusion

The results of the presented theoretical analysis can provide a good approximation for the prediction of the investigated gas-particle flow properties.

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